PSYCHOLOGY AND EDUCATION (2022) 59(2): 1454-1459
ISSN: 1553-6939

Limit Readiness Assessment of the First-Year Higher Education Students

Lulu Choirun Nisa [1] *, St. Budi Waluya [2] , Kartono [3] , Scholastica Mariani [4]
[1,2, 3,4] Semarang State University, Indonesia
*lulu.choirunnisa@gmail.com

ABSTRACT

For mathematics students, limit is the first subject encountered through Calculus. As the first material, several studies show that students get gaps in thinking about subjects related to competence in mathematics school and advanced mathematics. Therefore, readiness analysis is needed to determine the appropriate Limit learning design. The study was conducted on the first semester students majoring in Mathematics. The analysis results show that students' highest readiness is in the ability to count operations, integers and rational numbers, and algebraic operations. The most minimal competencies mastered by students to learn limits are in determining domains and range of functions and expressing distances in absolute value notation.

Keywords

Readiness Assessment; Limits; Mathematics Higher Education

Introduction

Limit is a concept that distinguishes Calculus from other branches of mathematics. The concept of limit is also central to studying of physics, engineering, and social science (Purcell et al., 2010). Meanwhile, according to Tall (1992), limit is a concept "which signifies a move to a higher field of mathematical thinking" (Juter, 2006). This opinion shows that the concept of limits is an essential part of the basis of mathematical analysis. Students who need help understanding the concept of limits can have problems dealing with concepts such as convergence, continuity, and derivation. The concept of limit is also connected to many other mathematical concepts, such as functions and infinity.

On the other hand, if an individual understands the concept of limit, then the connected concept becomes easier to work with. However, students need help understanding the concept (Juter, 2006). In Sweden, for example, the starting material for university mathematics usually consists of Algebra and Calculus, including Limits, which have proven difficult for students to learn (Cornu, 1991; Tall, 1991; Tall & Vinner, 1981) - (Juter, 2006). Likewise, in Indonesia, limit is the first Calculus material obtained by students majoring in Mathematics or Mathematics Education in the first year.

Learning limits takes more time and effort than any other Calculus material (Juter, 2006). Many mathematicians have worked on the subject of Limits over different periods and with different approaches for hundreds of years with obstacles that took a long time to overcome. These studies provides an overview of the complexity and characteristics of the limit function concept.

Pre-calculus concepts such as functions and solving equations are needed to master the limit concept. Even so, many students need a more robust understanding of this concept, which hinders the achievement of Calculus in the first year (Kay & Kletskin, 2010). Several international studies have noted that students have difficulty understanding the types of mathematical concepts required to succeed in advanced mathematics. Jourdan, Cretchley, & Passmore (2007) reported that students experienced significant challenges in algebra, functions, and trigonometry. Lawson (2003) compared diagnostic scores from 1991 to

2001 and found a significant decline in competence in most areas of pre-calculus mathematics tested. A recent Canadian report claims that current students need more proficiency in introductory algebra, trigonometry, exponentials, and logarithms (Kay & Kletskin, 2010). The university has also observed a significant decline in student performance (Ontario Ministry of Education, 2006).

The expected learning outcomes in the Limits material are to understand the definition of limit, to determine the limit value using the definition, to apply the limit theorem to solve limit problems, to determine one-sided limits, and to determine the relationship between limits and the continuity of functions in an interval..

From a historical perspective, the concept of limits has developed for hundreds of years. Leibniz and Newton worked in the 16th century to develop theories to solve problems of the area and extreme values in different notations but with the same essence. However, other mathematicians criticized their study for not being able to answer some questions, such as a clear definition of limits. These critics brought difficulties to the development of his theory. Until then, Weierstrass presented a strict definition of limit, that is, A is called the limit of $f(x)$when $x \to a$, if for every $\varepsilon > 0$there exists $\delta > 0$such that $0 < |f(x) - A| < \varepsilon$for every $x$in the domain with $0 < |x - a| < \delta$.

The definition of limits causes some difficulties in the teaching and learning process. Several studies show that:

a. A gap appears in the cultivation of the concept of limit, namely the gap between students' perceptions of limits as processes and objects, limits as infinite and finite, and limits as intuitive and formal (Juter, 2006).
b. Apart from the complexity of the content, learning difficulties of limits may arise from cognitive gap problems between the definition and the actual concept of limits. Students can remember definitions but still need to understand them (Vinner, 1991). According to Vinner, the mathematical definition represents the conflict between the mathematical structure and the achievement of concepts through cognitive processes.
c. Students have their conceptions of limits, namely (Denbel, 2014):
    - Students see limits as unreachable
    - Students see limits as an approach
    - Students see limits as limits
    - Students see limits as dynamic processes and not as static objects
    - Students get the impression that a function will always have a limit
d. At first, students can state that they understand limits intuitively. It will create a feeling of control and think that they know about the concept, even if they fail to complete tasks that require greater mastery of the definition (Cornu, 1991).
e. Students view the word limit more precisely than the word "approach" or "approach"; and have problems with some mathematical expressions about limits and confusion (Denbel, 2014; Monaghan, 1991)
f. Another difficulty is with the “for each” and “there are” quantifiers in the definition. The meaning of quantifiers in everyday life is slightly different from those used in mathematics. Students conceptions of ideas before learning often block the entry of mathematical conceptions. Students who solve problems tend to use these spontaneous conceptions rather than mathematics.
g. Students struggle to make mental schemas because of formal definitions (Juter, 2006). This difficulty occurs in the transition to advanced mathematics, where the concepts of limits have an intuitive basis based on experience, to concepts where they are determined by formal definitions and their properties, reconstructed through logical deduction (Denbel, 2014). This gap produces various possible problems in studying limits, which must be assisted with learning tools to bridge these problems.

Literature Review

Calculus Readiness

In order to begin their study of Calculus, students must possess a strong foundation in working with functions and the ability to perform algebraic manipulations. These competencies, however, often need to be improved in the first year and impede students' efforts in advanced mathematics (Kay & Kletskin, 2010). One of the critical problems noted was that a severe lack of essential skills hampered students in mathematics courses from doing algebraic calculations and manipulations fluently and accurately (Kay & Kletskin, 2010). Five significant skills necessary for any students taking calculus: Algebra Manipulation, Solving Algebraic Equations, Graphing and Evaluating Functions, Simplifying and Solving Exponential and Logarithmic Equations, Evaluating and Solving Trigonometric Functions.

Students already have study limits at schools for different purposes. Learning outcomes of limits in school mathematics are explaining the limits of algebraic functions (polynomial functions and rational functions) intuitively and their properties and determining their existence. While in higher education, students need to understand the definition of limits, prove the value of limits with definitions, and prove the existence of a limit. There is a difference in the competence of graduates between school mathematics and advanced mathematics. As new students with various origins and elemental abilities, it is necessary to test their competency readiness before getting the content of limit and calculus. Therefore, it is necessary to do a readiness test for students to identify the material and competence that must be strengthened before limit learning is given.

Limit Readiness

$\lim_{x\to c} f(x) = L$means that for every $\varepsilon > 0$(however small), there is$\delta > 0$ such that $|f(x) - L| < \varepsilon$with the provisions $0 < |x - c| < \delta$. In other words, $0 < |x - a| < \delta \Longrightarrow |f(x) - L| < \varepsilon$.

The definition of limit contains several mathematical concepts: numbers, functions, absolute values, inequalities, implication logic, and function operations. In more detail, the indicators of each competency needed by students in order to understand and apply the definition of limits are:

1. Operations on integers and rational numbers. his concept is primarily needed to understand the concept of a tiny number which in the definition of the limit will ε and δ.
2. Algebraic operations, are needed in performing function operations, mainly when operated with L values, in the notation $f(x) - L$
3. Determine the number that is close to a certain number. These competencies are needed to understand the notation$x \to c$
4. Specifies the value of a function, which is required in identifying the approximation value $f(x)$.
5. Provide the concept of range in other representations, such as number lines and absolute value notation. This competency is needed to help read function graphs and understand the meaning of $|x - c| < \delta$ and notation $|f(x) - L| < \varepsilon$.
6. Solve the solution of the inequality. The definition of limit contains several inequalities, which to perform algebraic operations and solve them first requires skills to solve equations.
7. Determine the absolute value inequality. The definition of limit also contains some absolute value inequalities, which to do so first requires skills to solve absolute value equations.
8. Determine the mathematical logic of implications necessary to understand the relation between sufficient and mandatory conditions in the definition of limit.
9. Determine the range and domain needed to determine the position and existence of c and L.
10. Reading function's graph is necessary because showing the value and existence of the limit requires visualization of the graph.
11. Create a graphic sketch. In situations where students only have information about $f(x)$, students need the skill of sketching graphs of known functions for limit visualization.

Methodology

This study was conducted in August 2022 with the research subjects of 30 first-semester students majoring in Mathematics Education at Walisongo State Islamic University, Semarang, Central Java, Indonesia. The data was taken by using test and interview methods. The test was given before the students received the Calculus material. The instrument used is a test sheet referring to the prerequisite material described in the Limit Readiness chapter. The test results are suspended with the following conditions:

Score 0: students can not answer at all or answer but out of context

Score 1: students answer according to context but make mistakes in solving

Score 2: students answer concepts and calculations correctly

Interviews were conducted to confirm students' answers. The data were analyzed using quantitative and qualitative descriptive approaches.

Results

The recapitulation of the work of all students is presented in the following table.

No	Indicator	Score		
		0	1	2
1	Integer operations	30%	3%	67%
2	Rational number operations	23%	7%	70%
3	Algebraic operations	30%	7%	63%
4	Determine the number that is close to a certain number	37%	43%	17%
5	Determining the value of a function	77%	10%	13%
6	Declare range/distance in absolute value	90%	10%	0%
7	Determining the solution of a linear equation	33%	10%	57%
8	Finding the solution to a linear inequality	50%	10%	40%
9	Determine the solution to the absolute value equation	73%	0%	27%
10	Determine the solution to the absolute value inequality	80%	7%	13%
11	Determining the implication truth value	90%	0%	10%
12	Define range	97%	3%	0%
13	Define domain	100%	0%	0%
14	Reading function graphs	87%	3%	10%
15	Sketching graphics	93%	3%	3%
16	Knowing the undefined function	90%	0%	10%

Based on the table above, the competencies that students must master in order from the highest to the lowest percentage are as follows:

No	Indicator	Percentage Score 2
1	Rational number operations	70%
2	Integer operations	67%
3	Algebraic operations	63%
4	Determining the solution of a linear equation	57%
5	Determining the solution of a linear equation	40%
6	Determine the solution to absolute value inequalities	27%
7	Determine the number that is close to a certain number	17%
8	Determining the value of a function	13%
9	Determine the solution to the absolute value inequality	13%
10	Determining the truth value of the implication	10%
11	Reading function graphs	10%
12	Knowing the undefined function	10%
13	Sketching graphics	3%
14	Declare range/distance in absolute value	0%
15	Define range	0%

16	Define domain	0%

In the table above, it can be seen that there are only four indicators controlled by more than 50% of students, namely performing operations on integers, rational numbers, algebraic operations, and solving linear equations. While on the other 12 indicators, students who can complete it correctly are less than 50%. There are three indicators where the students answered incorrectly: stating the distance in the form of absolute values and determining the range and the domain.

On the indicator of performing arithmetic operations, students performed better on tasks involving irrational numbers than integers. The questions used to test the operation of rational numbers are $1-\frac{1}{1+\frac{1}{2}}$, while the questions used to test the ability to operate on integers are $5[(16:2)+(-5\times 2)-2(-3-4)]$. The interview results show that errors in integer operations are due to calculation errors and inaccuracies in seeing brackets. Meanwhile, the failure to complete the calculation of the arithmetic operations of rational numbers occurred due to two things, namely (1) arithmetical errors and (2) not mastering the concept of division by fractions.

On indicators of algebraic operations, students complete $\frac{(x^4y^{-3})^2}{x^3y^5}$. 63% of students can complete it correctly, and the rest need help to do well. Most of the student's error lies in the operation of the power variable.

In the indicator of solving linear equations, students are asked to find a value $x$that meets $6.4x-3.2=4.4x+1.2$. Only 17 out of 30 students can solve this equation. Three students experienced errors in calculating, and the rest could not answer.

Apart from the four indicators above, more than 50% of students need help to answer correctly. Only 40% of students can complete to completion in the indicator of determining linear inequalities. There are 10% or three students who work with the correct concept but need to correct the calculation. While the remaining 15 people could not work with the correct concept where the mistake made was changing the inequality notation into an equation at the time of completion. The interview results stated that they understood the meaning of the inequality notation > or <, but in solving it, they used to change it into equation form first.

In the readiness test for the absolute value equation, students are asked to complete $|2x-1|=6$. The expected answer is that students can solve the equation and find the value of x.

In the readiness test on absolute value inequalities, students are asked to complete $|x-3|\leq 4$. The expected answer is that students are able to parse $|x-3|\leq 4$into $-4\leq x-3\leq 4$, but only 13% or four students can complete it correctly, while the rest do not answer or solve using ordinary inequalities, which means eliminating the meaning of absolute value notation.

Specifically, on the indicator determining a number close to a certain number, 43% of students state that the number approach to 2 is 3. For this answer, the researcher gave a score of 1 to indicate that in students' minds, the concept of numbers refers to integers. The interview's result reinforces this test result in which the researcher asks students to give another number close to 2, and the student mentions 1.

Other unpreparedness shown by students in the competence to determine the value of implications, read function graphs and identify values x that make a function undefined. In these three indicators, only three students can complete it correctly, each of whom is a different person.

The students' unpreparedness to learn the most prominent limits is in material related to domains, ranges, stating distances in absolute values, and sketching graphs, indicated by the achievement below 10%.

The readiness test also asks about students' perceptions of limits. Of the 30 respondents, there are five types of perceptions about limits:

a. Limit as a boundaries of a function (11 students),
b. Limit as a point that makes the function undefined or infinity (7 students),
c. limit as a function value approach at a point (2 students),
d. limit is one of the characteristics of a function (7 students), and
e. other said that they do not know what the limit is and have no idea what the limit is (3 students).

The four types of perceptions above align with the findings of Denbel (2014), i.e., that students see limits as something unreachable, as an approach, as a limit, and the impression that a function will always have a limit (as a function characteristic). The student's perception found by Denbel but not appearing in the subject of this research is that the limit is a dynamic process, not a static object.

Discussion

The analysis of student work results shows that of the 16 competencies required to study Limits, only four competencies are mastered by more than 50% of students, namely rational number operations, integer operations, algebraic operations, and determining linear equations. These four competencies are related to numerical abilities, which support students in learning the definition of limits. While the competencies that are directly related, such as determining the number stating the distance in absolute value notation, solving absolute value inequalities, and understanding the implication truth value, more than 50% of students have not mastered it.

This gap will impact the learning design of limit, where the compiled genetic decomposition must consider student competency deficiencies. Genetic decomposition, according to Piaget, describes the composition of knowledge and how knowledge develop (Nisa et al., 2022). Therefore, to build students' conceptual construction on limits, the decomposition should begin with the construction of prerequisites that students have not mastered, according to the results of this study.

Based on the results of interviews, some of the obstacles stated by students in completing this readiness test were because: (1) they forgot because this readiness test was carried out in their first week as students, and there was a long time they did not study since they were in high school; (2) miscalculation; (3) the mindset of integer numbers is more robust than real numbers, so we see numbers that are close to specific numbers as the closest integers.

References

Cornu, B. (1991). Limits. In *Advanced Mathematical Thinking* (pp. 153–166). Kluwer Academic Publisher.

Denbel, D. G. (2014). Students ’ Misconceptions of the Limit Concept in a First Calculus Course. *Journal of Education and Practice*, *5*(34), 24–41.

Juter, K. (2006). *Limits of functions: University students’ concept development: Vol. PhD*.

Kay, R., & Kletskin, I. (2010). Evaluating the Use of Learning Objects for Improving Calculus Readiness. *Journal of Computers in Mathematics and Science Teaching*, *29*(1), 87–104.

Monaghan, J. (1991). Problems with the Language of Limits. *For the Learning of Mathematics*, *11*(3), 20–24.

Nisa, L. C., Waluya, S. B., Kartono, & Mariani, S. (2022). Developing Mathematical Conceptual Understanding Through Problem-Solving: The Role of Abstraction Reflective. *Proceedings of the 6th International Conference on Science, Education and Technology (ISET 2020)*, *574*(Iset 2020), 38–42. https://doi.org/10.2991/assehr.k.211125.009

Purcell, E., Varberg, D., & Steven, E. R. (2010). *Calculus (9rd Edition) - Dale Varberg, Edwin Purcell and Steve Rigdon.pdf* (pp. 215–243).

Vinner, S. (1991). The role of definitions in the teaching and learning of mathematics. In *Advanced Mathematical Thinking* (Issue January 1991). https://doi.org/10.1007/0-306-47203-1