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Review Article

Analysis of Infinity Conceptions in Mathematics among graduate, undergraduate and secondary Jordanian students

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Abstract

This study investigates students' conceptions of infinity in mathematics, including graduate students enrolled in mathematics education program, undergraduate mathematics students, and secondary students. Two issues were of concern to this study: conceptions of infinity and differentiation between large finite sets and infinite sets.

Using quantitative-qualitative survey approach, the results revealed that students possess different conceptions of infinity that are based on both potential and actual infinity. Moreover, students faced difficulties in understanding infinity, and for those who showed understanding of infinity concept did not apply their experience in the formal education of set theory and calculus courses that they have been studied. Several implications of the results could proceed to overcome epistemological and didactical obstacles related to the infinity concept in order to improve students' ability to deal with situations in their mathematical courses.

Keywords: Potential infinity, actual infinity, infinity in mathematics, mathematics education, finite and infinite sets.

Introduction

The concept of Infinity has played an important role in many branches of human knowledge; philosophy, theology, science and mathematics (Nunez, 1994, 2004). Russel (without date) argues that the nature of infinity belonged in former days to philosophy but belongs now to mathematics. The interaction of philosophy and mathematics is seldom revealed so clearly as in the study of the infinite among the ancient Greeks (Furley, 1982; Knorr, 1982).

The concept of infinity as an intuitively contradictory concept engaged the minds of philosophers and mathematicians, and took two different meanings; potential and actual, that differentiate the ways of looking at infinity as a process or as an object. So, according to Aristotle "potential is a sequence of actions that continues without end", while the actual infinity as "the not finite that is presented in a moment of time"(Maria et al., 2010, 1771). Unlike Broitman (2013) argued that Descartes did not recognize the existence of potential infinity in mathematics, moreover, he denied any potential or actual existence of an infinite number and did not consider the infinite in his mathematics.

In the meanwhile, Leibnez rejected infinite number, but he distinguished between the "collection of parts that form a single thing, and the collection of parts that cannot form a single thing" (Harmer, 2014, 236). In addition, Bolzano contribution in the field of infinity was defending infinity as a totality, using large finite set as an example (Kolar & Codez, 2012; Voskoglou, 2016). In the 19th century and moving forward to Cantor, Cantorian set theory introduced the best contribution of actual infinity to the foundation of different mathematical systems, where the treatment of infinity by Cantor built a theory of different sizes of infinity with definitions of cardinal and ordinal numbers (Maria et al., 2010). Also, the problem of infinitesimals was put on a mathematical basis by Robinson (Wijeratne & Zazkis, 2016).

Tall & Tirosh (2001) argued that individuals move between different aspects of infinity, and imagine infinity as a process. Regarding advanced mathematical thinking, Tall (1992) argued that in mathematics there are at least three forms of actual infinity: "cardinal infinity, ordinal infinity and nonstandard infinity" (p.505). For learners, they usually define infinity as "something that continues and continues", " it is not a complete entity" (Tirosh, 1999, 344). Learners also explain infinity as the limit notion, whereas, they explain actual infinity referring to large finite numbers (Cornu, 1991)

Tall (2001) categorized the concepts of infinity as natural and formal infinities, where the formal form of infinity starts with the Peano postulates as a formal definition for the natural numbers, where this extends to the formal concepts of infinite cardinals and ordinals, and then shows how formal ideas of infinitesimals can be interpreted in a visual manner as points on number line.

The term infinity means different things in different contexts. It might be potential infinity which represents a process that is finite but could go on infinity in the sense of cardinal infinity of Cantor (Arzarello et al., 2004). Aristotle argued against the actual infinite, and he considered the potential infinite, his idea was that we cannot consider the natural numbers as a whole; however they are potentially infinite in the sense that given any finite collection we can always find a larger finite collection (Maor, 1991). Later Greek mathematicians were affected by Aristotle idea of potential infinity, Euclid, for example, proved that given any finite collection of prime numbers there must be a prime number not in the collection, which means that there are infinitely many prime numbers (Miller, JR, 1982; Siegmund-shultze, 2014)

In mathematics, infinity is defined in the context of set theory, the word infinity comes from the latin infinitas or unboundedness. The mathematical form of infinity are the cardinal and ordinal infinities of set theory, where Dedekind and Cantor adopted the idea of one to one correspondence as a standard for comparing the size of sets and to reject the Galileo approach that the whole cannot be the same size as the part. That is to say, an infinite set is defined as one having the same size as at least one of its proper subsets (Kanamori, 2009).

From a different perspective, potential infinity is characterized by a process that could be repeated without end, where it occurs often in mathematics such as a sequence of regular polygons that are drawn inside a circle with more and more sides without end (Nunez, 2004, 2005). If we take into account all the numbers which belong to a segment of the real straight line, it is infinite numbers, this infinity is the actual infinity which is different from the potential infinity, and exactly the idea that Cantor adopted (Bussoti, 2014). Also, it is the kind of infinity to which Augustin spoke of the "whole of number", and to which Hegel spoke of "good infinity" against "bad infinity" (Livingston, 2014).

According to the previous perspectives about infinity concept, it is clear that mathematicians played a fundamental role to produce a clear distinction between potential infinity and actual infinity. Considering the curriculum of secondary students in Jordan, infinity is dealt in calculus course without giving a definition of the concept, while regarding the undergraduate courses, set theory is a core course for those who specialized in mathematics, where the different ideas are dealt in a formal approach.

Review of Literature

Many research studies related to mathematical infinity concluded that infinity is a difficult concept to be understood and specially the actual form of it (Voskoglou, 2016; Sokratis, 2017). Most of the research studies on infinity were in the potential and in the actual sense. In this context, Moreno & Waldegg (1991) investigated the potential and actual uses of the term infinity, where it is considered at times as an adjective and at times as a noun, and they added that the acceptance of potential infinity produced a way of thinking to develop an actual conceptualization of infinity. Ucar et al., (2015) concluded that the middle school students can construct an understanding of potential infinity, but they cannot understand actual infinity, while Sbaragli (2006) pointed out that previous research revealed inconsistency to deal with actual infinity by university students.

Many research articles aimed at understanding the difficulties students have to face, in different learning stages. Epistemological and didactical obstacles were raised as results of the studies in this context. On this respect, Aztiken et al., (2010) pointed out through the review of literature that research revealed an epistemological obstacle that faces students referring to the acceptance of correspondence between the set of natural numbers and its subset of even numbers. In the same context, Fishbein (2001) observed another phenomena in the context of geometry and numbers, where different students of high secondary schools marked that there are more points in a longer segment than in a shorter one, and since the set of even numbers is a subset of the natural numbers set, the former seems to be formed of a smaller number of elements. This misconception highlights students' belief that the whole is greater than its part both for finite and infinite. The two previous misconceptions are based on the overgeneralization to infinite sets of what has been learnt by students of finite cases.

One of the major components of research in mathematics education has been the study of students' conceptions and reasoning about infinity, as an advanced mathematical concept. Monaghan (2001) reviewed and analyzed different articles that discussed views of infinity of pre- university students, and concluded that students view infinity as a process and as an object, the contradictory nature of infinity; where the idea that something could have no limit is conflicting and unreasonable for students, and type of tasks and its context influenced the nature of responses of students about infinity. In the same context, Dubinsky et al., (2005) argued that potential infinity could be described by the process of creating as many points as desired on a line segment to account for their infinite number, whereas actual infinity would describe infinite number of points on a line segment as a complete entity.

From a cognitive viewpoint, Nunez (1994) proposed to understand the development of the notion infinity among 32 students of different ages with high and low intellectual- academic performers. An opened task was used to collect data by individual interviews. The results showed different categories of responses, where 66% of the older students said that the process of subdivision a distance comes to an end, while less than 25% said that it is possible that the process will continue endlessly. On the contrary, Nunez (2004) concluded that "the nature of infinity can be understood in terms of human ideas and human cognitive mechanisms beside formal logic" (p.68).

Regarding teachers' conceptions about mathematical infinity, Sbaragli (2006) revealed that primary school teachers possess specific misconceptions about mathematical infinity, and they have total lack of knowledge about this topic. In the same context, the study conducted by Maria et al., (2010) revealed that the majority of elementary teachers comprehend infinity as a continuous and endless process and they faced difficulties and hold misconceptions about the two aspects of the infinite concept as a process or as an object. Moreover, Kolar & Cadez (2012) concluded that students- teachers' understanding of infinity depends on the type of task and on the content of the task, and when solving tasks related to infinitely large and infinitely many, their justifications based on actual infinity, while they use arguments based on potential infinity when solving tasks of the type infinitely close.

In the same context, Celik & Aksan (2013) investigated the perceptions of pre-service mathematics teachers on the concepts of infinity, indeterminate and undefined. Based on the data analysis, the candidate teachers (23%) has been more successful in explaining the concept of indeterminate.

Through a philosophical-mathematical inquiry into the concept of infinity, Kennedy (2007) posed an inquiry question; "is infinity a number?" this question revealed different conceptions among students, where some thought that infinity is a number, others that it was not, and after a series of discussion, students' infinity conception came to be an expanding process, and is not an actual number. Based on addressing inquiries, Singer and Voica (2008) analyzed how perceptions on infinity develop in students of different ages, and using the geometric context, students highlighted three categories of primary perceptions: processional, topological and spiritual.

Regarding university students' conception of infinity, Montoro (2005) showed that the way of teaching mathematics was the major variable for understanding the concept of infinity in favor of the formal teaching, followed by educational level, where advanced undergraduate students understand the concept of infinity better than the first year students. Mamalo & Zazkis (2008) investigated the conceptions of university students with different mathematical background; undergraduate students in Literal Arts programs and graduate students in mathematics education, the arguments of the respondents revealed that the students of both programs judged infinity as potential rather than actual. Eliciting constructs of (Ph.D) students concerning infinity and the effect of the set theory lessons to their constructs and infinity images, Aztikin et al., (2010) revealed that students generally have an understanding of potential infinity and after formal teaching they showed evidence of actual infinity.

As an integration between the concepts of infinity and limit, Kim et al., (2005) investigated the mathematical discourses of American and Korean students' thinking about infinity and infinite. The words used by the Korean students to express infinity and infinite were mathematical.

In the meanwhile, all students either American or Korean, with the exception of two undergraduate students who took a calculus course, did not get the formal meaning of infinity in mathematics. In order to overcome conceptualizing a limit as a potentially infinite process, Kidron & Tall (2015) conducted a study to investigate the convergence of a sequence of functions visually in order to lead to the formal definition of the limit concept, the results revealed that half of the participants argued that the infinite sum of the functions was regarded as an object and not as a process.

Problem and Research Questions

In Jordan, concept of infinity has no explicit guideline and no formal education either in secondary mathematics curriculum or undergraduate mathematics syllabus but depends on the teachers' and faculty members' understanding of the concept. To meet the needs of graduate , undergraduate and secondary students in Jordan, one way is to evaluate and analyze their conceptions of an abstract and advanced mathematical concept in order to achieve different purposes, including; providing feedback and evidences on infinity understanding of students to faculties and secondary mathematics teachers for further improvement of formal teaching of this concept, and providing information to students about their misconceptions in order to consider in their studying to be more serious, specially most of the graduate students are mathematics teachers, several of the undergraduate students will be mathematics teachers, and many are planning to be graduate mathematics students. So, infinity conceptions of secondary and university students are studied, and information should be gathered in order to devise more systematic instruction regarding this concept. In the meanwhile, no studies are conducted at any students' educational level in Jordan that deals with the concept of infinity

The purpose of the study was to answer the following two questions:

1. What are the conceptions of infinity among Jordanian students? Do those conceptions differ according to the educational level of students?
2. Do students differentiate between finite and infinite sets.

Study Sample

The present study involved (309) participants; (120) secondary students, (150) third and fourth year undergraduate students who are mathematics major, and (39) graduate students who are enrolled in mathematics education program; (180) females and (129) males. Some of the graduate students are mathematics teachers. Secondary students were selected by random clustered method from three secondary schools; one is located in the south of Jordan, one in the north and the third in the middle. The undergraduate students were selected by random clustered method from five public universities that are spread in the north, south and the middle of Jordan, while the graduate students, who are enrolled in master program or Ph.D program in mathematics education, were selected as an available sample from two public universities.

Instruments and data collection

Data were collected through mathematical tasks about infinity which took two sessions to complete by the secondary students, each with 45 minutes, while it took 60 minutes to complete by the undergraduate and the graduate students during one of their regular lectures. The respondents were given the tasks at the same time and were completed individually. Two tasks were used, the first included two subtasks that asked for the meaning of infinity in mathematics, writing expression or statement that includes the word infinite and writing expression that includes the word infinity. The second task asks to differentiate between finite and infinite sets; it includes eleven arithmetical and geometrical sets. The tasks used in this study were prepared after reviewing many related articles such as: Tirosh (1991); Boero et al., (2003); Monaghan (2001); Kim et al., (2005); Sbaragli (2006); Pehkonen et al., (2006); Aztiken et al., (2010); Maria et al., (2010); Kolar & Cadez (2012); Ucar et al., (2015).

Data analysis

This study is a quantitative - qualitative survey, where proportions were used to analyze the data quantitatively, and the induction-deduction and thematic approaches (McMillan & Schumacher, 2001) were used to analyze data qualitatively in order to answer the different questions. In addition, reliability of data analysis was tested by inter-rater and intra-rater reliability through analyzing a sample of the responses that belong to each research question by two judges.

Results and Discussion

As a matter of fact after a first reading of the tasks, the respondents from all different educational levels; graduate, undergraduate or secondary, were embarrassed, they asked what shall we write? we do not know what to write, we never faced these questions, how can we reason our answers? but finally they decided to write what comes up to their minds, even if it is not reasonable. However, it was stated clearly from the beginning that their names would not appear in their answer sheets.

Results of question 1: what are the conceptions of infinity among Jordanian students?

In order to answer this question, two open questions were introduced to the participants; what does infinity in mathematics mean to you? write a statement that includes the word infinite and another statement that include the word infinity.

Thematic approach was used to analyze the written responses of the students at all levels (graduate, undergraduate, secondary), where each response of each respondent was induced regarding a group of indicators, then the responses were classified into different themes. These themes show aspects that indicate to the conception or to the way of understanding the meaning of the concept of infinity. The themes include infinity as indefinite, infinity as a positive large number or a negative small number, infinity as unlimited, infinity as limit concept and infinity as a process. Following are some examples of the written comments by students according to their educational levels (graduate (g), undergraduate (ug), secondary (s)).

For infinity as indefinite, following are some examples:

" Something that cannot be expressed as a number or a quantity"; "something that does not exist"; "something without borders"; "cannot be measured"; "cannot be counted. (g)"

"Something without border", "an object that does not exist". (s)

"Unknown value either positive or negative". (ug)

For infinity as a positive large number or as a negative small number, following are some examples:

"A very large number that describes the behavior of functions, sequences and series"; "very small number that cannot be reached";

"quantities that increases or decreases and reaches a very large or a very small quantity that cannot be noticed" (this last response was described by a graph of a function); "the largest number"; " very large number that cannot be reached, and expressed by the symbol ∞"; " very large positive number, very small negative number"; "absolute large number (+∞) greater than any given large number, or very small number (- ∞) smaller than any given small number". (g)

"Very large number (unimaginable)"; "a very large number with no limit"; "very large unknown number"; " very large number without border". (ug)

For infinity as unlimited, following are some examples:

" Something that has no limit, unlimited such as integer numbers, real numbers"; "unlimited such as number line"; something that has beginning point without end point"; " a point in the space where its location is unknown". (g)

"Unbounded object"; "unbounded number"; "boundary that cannot be reached". (ug)

"Life is unlimited"; "life has no end"; "an object that has a beginning point without end point"; "an object without a beginning point and without an end point". (s)

For confusing with the limit concept; a Ph.D student described infinity as a *"limit of a function at a given point", while a secondary student wrote: "a function approaches a value at a point but do not reach it", and an undergraduate student wrote: "infinity is a behavior of a function that approaches infinity"*

For infinity as a process: few students express infinity as a process, such as*: "when you count, you will never stop counting"*

The previous responses clarify that for line, half line (ray), plane, space, infinity and unlimited seem to be synonyms. Whereas, in the cases of numbers and points, the respondents refer to infinity as a very large finite number or infinite number.

Regarding the use of infinity as an adjective (infinite) and as a noun (infinity), the majority of the students of the different educational levels gave statements using the term infinite as an adjective, and the term infinity as a noun. Using the term as an adjective or as a noun was concentrated on either mathematical or experiential examples. The mathematical statements can be classified as: sets of numbers (e.g. real, natural, rational, irrational, integer, even); geometrical statements (e.g. set of lines, segments, half of line); numbers that can be expressed as infinite sequence of decimal digits(e.g. 1/3= .333…); sets of numbers within closed, opened or half opened intervals (e.g.]1,4[, [0, ∞],]-2,9]); functions (e.g. exponential function is infinite). The experiential statements were related to sunrays, life, stars and universe. The following are examples of the respondents' expressions.

"Science is infinite"; "infinite thinking"; "universe is infinite"; "infinite is adjective of something without end"; "my hope in life is infinite"; "numbers are infinite"; "lim(f(x) is infinite"; "numbers goes to infinity"; "life lasts to infinity"; "universe continues to infinity". (s)

"Set of real numbers is infinite"; "even numbers are infinite"; "rational numbers are infinite"; "there are infinite numbers of fractions"; "number of stars in sky is infinite"; "infinite is a description of something but not a value"; "limit of a function may be infinity"; "some equations in mathematics has infinite solutions"; "we cannot reach infinity in real numbers"; "set of real numbers =]-∞,∞[, limit of 1/x=∞ when x approaches 0". (ug)

Most of the statements given by the graduate students were related to the sets of numbers and points on segment or line, either when using infinity as adjective or as a noun. Moreover, few statements include: *"ray is infinite"; "exponential function is infinite"; "folding a paper without stop is infinite"; "number of lines that could be drawn from one point are infinite"; " number of lines that intersect another line at one point are infinite". (g)*

In addition to the previous responses, there were few responses that draw attention, for example; one of the undergraduate students wrote: "*assuming that 1/0= ∞, this means that 1= 0×∞ which is impossible, so infinity (∞) is undefined*". It is clear that this student tried to prove that infinity is undefined. A graduate student gave an informal definition of infinity by writing that " *infinity is a number that any number can be divided by to get zero", another graduate student , who is a mathematics teacher, argued that "infinity is the slope of a vertical line on the coordinate plane*", in the meanwhile, a secondary student connected infinity by counting the elements of a set where the counting process never ends.

Comparing these results with that of the international research (Kim et al, 2005; Sbargali 2006; Singer & Voica, 2008; Maria et al, 2009; Kolar & Cadez, 2012; Ucar et al., 2015), the themes of the responses in this study regarding conceptions of infinity show some aspects of international students' approaches to infinity. Although the students participated in this study have studied a calculus course, in addition to a set theory course within the undergraduate program, it is concluded that few of them had full appropriate conception of infinity in the mathematical sense, and most of them believe in the potential- actual duality of infinity in mathematics. Partly, the results of this study are consistent with the results of the international research in this context.

Regarding the use of the notions of infinity and infinite in sentences, most of the respondents especially the undergraduate and the graduate, used these terms mathematically with symbolic representations, also they mastered the adjective - noun duality through the statements introduced by them, since the two terms are part of the respondents' mathematical experience. For the secondary students, infinity and infinite were used mostly in the context of real-life phenomena. In addition, the descriptions of the undergraduate and graduate students were more rigorous than that of the secondary students when they were asked to give meaning of the infinity concept.

As Kolar & Cadez (2012) argued , the reasons for the difficulties that face students in all ages might be related to the abstract nature of infinity from one side, and to the contradiction between infinity concept and students' intellectual schemes which adapted finite realities around them from another side. Also, as Piaget claimed that since there is a close relationship between the nature of mathematical concepts and its development in individual's mind (Dubinsky et al., 2005), one of the reasons that prevents understanding of infinity as an object in mathematics and not as a process , are didactical obstacles.

Results of question 2: Do students differentiate between finite and infinite sets?

To answer this question, the respondents were given (11) sets with different descriptions and asked to identify each one either infinite or finite. Descriptions of sets and percentages of students who answered infinite are shown in table 1. The percentages were classified according to the educational level of the respondents (graduate (g), undergraduate (Ug), and secondary (s).

Table 1 shows that most of the students differentiate between the infinite and the finite sets with large cardinality, such as sets 1 and 2. In the meanwhile , they do not distinguish, to a certain degree,

Table 1: Description of sets and percentages of students who answered infinite

N.	Description of the sets	Percentages of answers that a set is infinite		
		(g)	(ug)	(s)
1	The number of equilateral triangles, each with its sides longer than that of the previous one.	0.9	0.96	0.81
2	The number of all numbers that are used when counting	0.92	0.87	0.63
3	The number of isosceles triangles that have line segment of 5cm as one of their sides.	0.88	0.61	0.51
4	The number of grains of sand in a desert.4.	0.72	0.87	0.56
5	The number of points that are 5 cm distant from a point A.	0.69	0.65	0.63
6	The number of equilateral triangles, each with its sides shorter than that of the previous one.	0.53	0.21	0.44
7	The number of decimal points that can be formed for the number (0.87).	0.5	0.61	0.38
8	The number of regular polygons that can be drawn inside a circle where the number of its sides is continuously increasing.	0.59	0.39	0.44
9	The number of decimals between 1 and 2.	0.77	0.82	0.56
10	The number of points that are 5 cm or less distant from the point A.	0.6	0.82	0.59
11	The number of days required to cross the distance between two cities A and B when A is 20 km from B using the following rule: On the first day you walk half of the distance, on the second day you walk half of the remaining distance, on the third day you walk half of the remaining distance, and so on.	0.31	0.21	0.25

between very large finite and infinite sets, such as the description of set 4(the number of grains in a desert) which includes a large finite number of elements which cannot be physically counted, where graduate, undergraduate and secondary students marked set 4 as infinite with percentages; 0.72, 0.87, and 0.56 respectively. For sets 1 and 2, their descriptions were the easiest to recognize, where the first contains the idea of expanding equilateral triangles in a plane and the second contains the idea of infinitely increasing numbers, and both contain the idea of infinitely large. Whereas, the description of set 8 (the number of regular polygons with increasing sides inside a circle) was marked infinite with proportions: 0.59, 0.39, and 0.44 by graduate, undergraduate, and secondary students respectively, and although its description is similar to set 1, the proportions of those who answered set 8 as infinite were smaller. The reason might be that the regular polygons are assumed to be drawn inside a circle as a boundary, where the circle is the limit of this set. In addition, the most difficult to recognize were the descriptions of infinite sets that include diminishing and approaching a point, such as sets 6 and 11, where both imply the principle of infinitely close.

Trying to compare between the answers of the respondents regarding their educational level, the descending order of the sets according to students' percentages who marked the different sets as infinite were as follows:

Graduate (g): 2,1,3, 10, 4, 5, 9, 7, 8, 6, 11

Undergraduate (ug): 1, 2, 4, 10, 5, 3, 8, 6, 9, 7, 11

Secondary (s): 1, 2, 5, 4, 10, 7, 3, 8, 9, 6, 11

From the previous descending order, it seems that there is similarity in understanding the concept of infinity among the different educational levels of the respondents, where the proportions of sets 1, 2, 4, 5 and 10 were the highest, whereas, the proportions of sets 6, 7, 8 and 11 were the least for students of different educational levels. This result may be due to the fact that sets 6, 7, 8 and 11 are bounded.

It is clear from the previous results that the responses of the students were affected by the type of the task; for example in the case of equilateral triangles description that corresponds to real life situations in sets 1 and 6, the sequence of expanding equilateral triangles is accepted as infinite by 0.90 of the graduate students, by 0.96 of the undergraduate and by 0.81 of the secondary students. On the other hand, only 0.44, 0.21, and 0.53 of the graduates, undergraduate and secondary respectively believe that the sequence of diminishing equilateral triangles is infinite. It seems that the concrete description or physical situation of an abstract mathematical idea make the students to believe that the infinite set is finite, where in this case drawing smaller and smaller equilateral triangles inside an equilateral triangle will end at a point, or may approach a given equilateral triangle.

In the meanwhile, drawing regular polygons inside a circle where the number of sides is continuously increasing will end to a limit which is the circle, this led most of the students to mark set 8 as finite. This is aligned with the idea of difficulties related to physical representations of geometrical tasks on infinity that support the findings of the studies conducted by Tall (1999) and Kolar & Cadez (2012). In the case of set 10 (the number of points that are 5 cm or less distant from the point A), where the percentages of graduate, undergraduate and secondary students who answered infinite were 0.60, 0.82, 0.59 respectively, and in this case students may be able to think of an infinite number of points even though the set is bounded. Moreover, it could be taken into consideration that a set is uncountable, as set 5, which has uncountable infinite number of points.

The results for item 4 (Table 1) which was believed as infinite by 0.72, 0.87 and 0.56 among the graduate, the undergraduate and the secondary students respectively, explain that the students from different educational levels consider the large finite whole of grains in the desert as infinite where this whole cannot be reached and the number of elements in a large finite sets cannot be counted.

Conclusion and Recommendations

In this piece of research, two issues related to infinity in mathematics have been investigated: conceptions of infinity among graduate, undergraduate and secondary students, and the differentiation between large finite sets and infinite sets.

The results of this study revealed that graduate and undergraduate students whose major is mathematics and who studied a calculus course, beside secondary students faced difficulties in understanding infinity. For those who showed understanding of infinity, they did not apply their experience in formal education of set theory and calculus courses. In addition, students possess different conceptions of infinity such as, a process, indefinite, positive large number or negative large number and unlimited. On the opposite, two major misconceptions were indicated by the students; the large finite set is infinite and geometrical sets of the type infinitely close are finite. So, one major conclusion is that the easiest type of sets to identify as infinite were those with descriptions based on infinitely large, whereas the students had difficulties with descriptions based on geometrical tasks of the type infinitely close. This might explain that secondary school and university based instruction resist intuitive explanations of infinity adapted from their real life experiences, and which give the students different cognitive sense for the idea, consistent with the results of the international studies that were conducted on secondary students, university students, and pre-service and in- service teachers.

The main contribution of this study is an increased understanding of the conceptions of infinity among secondary and university students in Jordan. In this context, the results of the study could introduce new support to the results of the international studies about infinity. According to the results of this study and to achieve a better understanding of problems about infinity among graduate , undergraduate and secondary students in Jordan, a systematic instruction in content on infinity in mathematics should be developed with different learning activities that encourage thinking about infinity. Moreover, implications of the results could proceed to overcome the epistemological obstacles by using inquiry and problem-solving based instruction using tasks that manipulate the different aspects and contexts about infinity. Moreover, the study could open the door for new research in this area with different objectives, methodologies and different tasks for collecting data in order to achieve in-depth information, such as using interview and different approaches to analyze data, in order to understand the transformation from potential infinity conception stage to the actual infinity stage.

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